Turbidity, Color and Wavelength

In the Light of Goethe's Discourse on Color (*Farbenlehre*)

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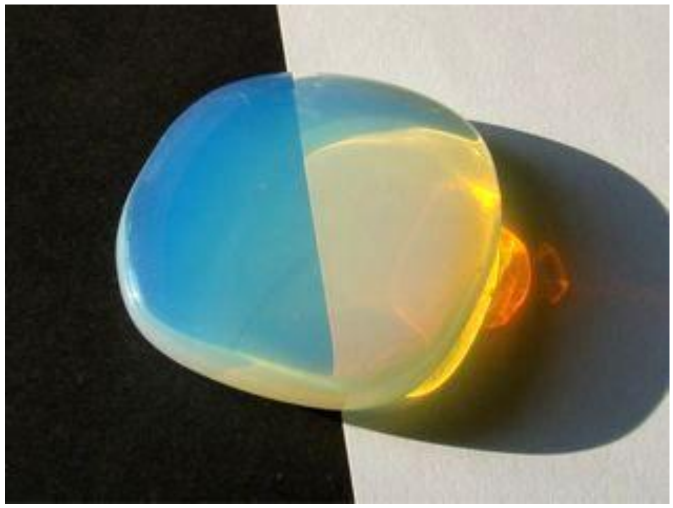

I. Introduction

In Goethe's treatment of color, the major emphasis is on brightness, darkness, and the medium between them. According to his observations that can be reproduced by anyone, color yellow appears when a bright white light is seen through a cloudy or hazy medium, and the color blue appears when an illuminated medium is seen in front of a dark background. He says[1]:

> We see on the one side light, brightness; on the other darkness, obscurity: we bring the semi-transparent medium between the two, and from these contrasts and this medium the colours develop themselves…

In this statement, Goethe introduces a concept regarding the medium that he will make use of repeatedly in his discourse – the *semi-transparent* medium – variously referred to as translucent, semi-opaque or even *turbid* medium. This term "turbidity" is the fulcrum on which his entire theory of color is developed, but Goethe, whose main focus was on the color phenomena themselves, did not provide a clear description of what this turbidity means physically, other than through examples. He was, naturally, criticized for this almost immediately[2]:

> He by no means understood the physical action of turbid media, but he made a great variety of experiments bearing upon this point.

In order to determine this physical action, a closer analysis is required, much as Goethe himself wanted. What can and cannot be called a turbid medium? What is the mode of action of this turbidity? The answers to these questions are not immediately obvious. Even more importantly, these questions themselves have to be clarified to make sure that they are logically consistent with the physical action being observed.

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[1] Goethe's *Farbenlehre*, No. 175

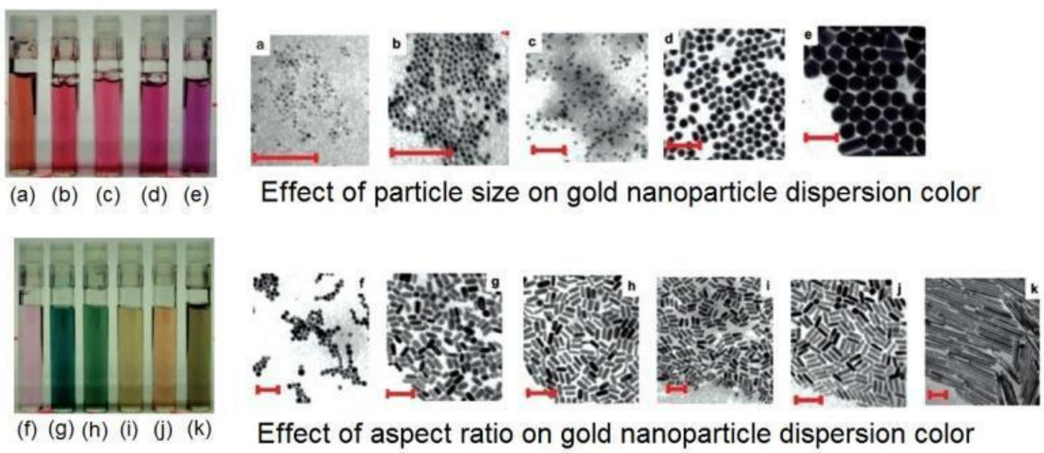
[2] John Tyndall, Popular Science Monthly, Volume 17 (June 1880), pg 220

II. "Atomizing" the Turbid Medium

The approach to questions of turbid or semitransparent media takes a specific path. For example, we have the statement from a comparatively recent physicist, George Wells[3]:

> Today every physicist knows that whether the colors are produced depends on the size of the particles in the medium.

Wells then goes on to describe the variation of color with the size of the suspended particles. It is also mentioned that cigarette smoke, which does look blue on a black background, looks grey near the mouth, and this is attributed to larger size of the particles due to moisture content near the mouth. This emphasis on the relationship of particle size to color can be seen in the scientific literature in microscope images such as the one below[4] where gold colloids are observed:



It is hence argued that the color that is seen depends on the kind of turbidity, which is determined by the geometrical structure of the particles which make it up. The blue or yellow color of the turbid medium is associated to the "scattering" of specific colors that depend on the size of these particles. In other words, the notion of light being a combination of colors out of which some are taken up, some are transmitted and some reflected, is applied on the microscopic scale to derive the presence of color in a turbid medium. Thus, several objections of this nature can be made, and are made, regarding the precise role of turbid media.

The primary issue that occurs as soon as one speaks of a turbid or semi-transparent medium is that there is usually an automatic focus on the size of the constituent particles. In fact, researchers who studied one of Goethe's own samples of semi-transparent media (GNF0074) which he had managed to get manufactured with great difficulty, attributed its cloudiness and action to silver particles embedded in the material[5]. The entire question is thus transferred to the microscopic realm. Whether these particles are dust, metallic impurities in a crystal, gold particles in a colloidal solution, or even the notion of "molecules of air", the entire picture gets "zoomed in" and the scene is shifted into the barely visible realm. The numbers of these particles are usually extremely large. In addition, interaction of light with matter is generally thought of as a kind of scattering and interference process at the microscopic level, with the macroscopic behavior determined by calculating the result of all the microscopic reactions. Each "particle" interacts with the others in various ways, making the number of interactions astronomically large once again. Hence, the phenomenon is "atomized" at two levels:

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[3] George A. Wells, Journal of the History of Ideas, Vol. 32, No. 4 (Oct. - Dec., 1971), pp. 617-626

[4] See https://ninithi.wordpress.com/metallic-nanoparticles/

[5] Thomas Nickol, Klaus-Jürgen Berg, Gunnar Berg, Goethes trübe Gläser und die »Beiträge zur Optik trüber Medien« von Gustav Mie, Goethe Jahrbuch 2008, p 204

1. In the search and focus on the constituent particles of the medium
2. In the varied interactions between these particles giving the resulting color

Since large numbers, of the order of several millions, is not uncommon for both these interactions, the actual calculation for a turbid medium becomes extremely intensive computationally. After Faraday's investigation of various colloids, this mode of analysis was begun in the mid-to-late 1800's by Tyndall, Rayleigh, and later C.V. Raman[6]. The most up-to-date theory used for calculating the action of a given turbid medium is the Lorenz-Mie Theory, developed mainly by Gustav Mie[7]. He assumed that light is incident on a sphere, and used electromagnetic equations involving the calculation of infinite series expansions to obtain the expression for the transmitted, absorbed and reflected light. Before the advent of faster computing, however, this was not practicable[8]:

> Understandably, prior to the development of electronic computers in the middle of the last century there were not many papers written on computing scattering problems using Mie's theory since the computational labour involved in evaluating functions such as Riccati-Bessel functions was quite extreme. Even with the rise of the computer it took some time before stable algorithms were developed.

Since the 1990's several programs such as MiePlot[9] have been written up in order to calculate the transmission of light. However, in spite of its immense complexity, it is said that:

> The Mie theory has restrictions when used alone. It does not allow for the influence of surrounding particles, which may cause re-scattering of the scattered radiation. Furthermore, it does not allow for the interference of the radiations scattering from various particles. This shortcoming is perhaps more serious than the former.

Hence, the most up-to-date approach in the physics of turbid media has partially addressed the first atomizing mentioned above, and has not properly addressed the second. It is very likely that this vast field of research that has been developed over the past century will be further extended in fine-tuning these calculations.

The question is, however: Is this a sensible path? By shifting the scene to the microscopic realm, quantitatively increasing calculations by the million and encoding that into software, the phenomenon as such is even less accessible to human experience and understanding. Since the method of questioning involves looking at the particle properties like size, shape, electric and magnetic responsivities, exchange goes like this:

Q: Why does solution A with particles of B have the color red?

A: Because, solution A is made up of B with size X and dielectric constant Y, so it transmits a wavelength Z.

Thus, a question on *color* is answered by describing spatial dimensions and electro-magnetic constants. However, a discrepancy may be spotted here, since the question asked in *one* domain of experience is being answered in *another* domain. This method is not an oddity, but is the extremely prevalent method of looking towards the atomic constituents of a substance for the explanation of its behavior. These can be small "billiard balls" or metallic spheres, or any other sort of matter. In other words, this method is a result of *atomism* as applied to light and color. It is at this juncture that the path outlined above was actually taken, from Faraday to Mie to the present day ideas on semitransparent media or colloids.

If atomism is the general approach to the turbid medium itself, what is the general approach to light?

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6 Milton Kerker, "Founding fathers of light scattering and surface-enhanced Raman scattering", Applied Optics, Vol. 30, No. 33, (1991) pg 4699
7 Gustav Mie "Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen", Annalen der Physik. 330 (3): 377–445 (1908).
8 Thomas Wriedt, Wolfram Hergart, The Mie Theory, pg 56, (2012).
9 http://www.philiplaven.com/mieplot.htm

III. Historical ideas on the nature of light

Light, which has been seen as the most ephemeral of phenomena for centuries, gained a new representation in the 17th century, when mechanical philosophers like Descartes, Gassendi and Isaac Newton treated light as a series of corpuscles or particles. The laws of geometric optics like reflection and the straight-line effects of light and shadow led to the use of the image of a small particle thrown like a mini-billiard ball or marble, bouncing off of surfaces.

Reflection and the image of its corpuscular counterpart.

Newton claimed[10]: *Are not the Rays of Light very small Bodies emitted from shining Substances?* Seeing light as a collection of small bodies clearly explains Newton's "decomposition" of light into colors using a prism, since a bag of colored marbles can be divided up very easily. If properties of a solid ball are assumed for light, and an experiment shows white light going into a prism and colors coming out, the conclusion reached is that white light is decomposed into the different colored rays. Hence, the idea of decomposition of light is *a concept of the solid imposed onto light*. According to Tyndall[11], Newton apparently emphasized that his theory of refrangibility did not depend on the corporeality (corpuscle nature) of light, However, in using the concept of decomposition the assumption is already incorporated into the theory, regardless of what one may state.

While the concept of "bouncing" in reflection may work well with the particle picture, refraction is not directly compatible with this – a marble normally doesn't go through a wall and get out the other side. Sure enough, inspired by the ideas of a Jesuit monk called Ignace Pardies, the Dutch scientist Christiaan Huygens picked up an explanation based on waves[12]:

> Refraction, how explained by Pardies. Compared with sound waves in the air, compared with water waves.

Hence, the attempt was now made to see light in terms of vibrations in fluids such as water and air. He explained refraction in these terms, and there is indeed some justification in this choice, since a simple experiment with water waves moving over tanks of different depths clearly shows a change of direction as it is seen in refraction.

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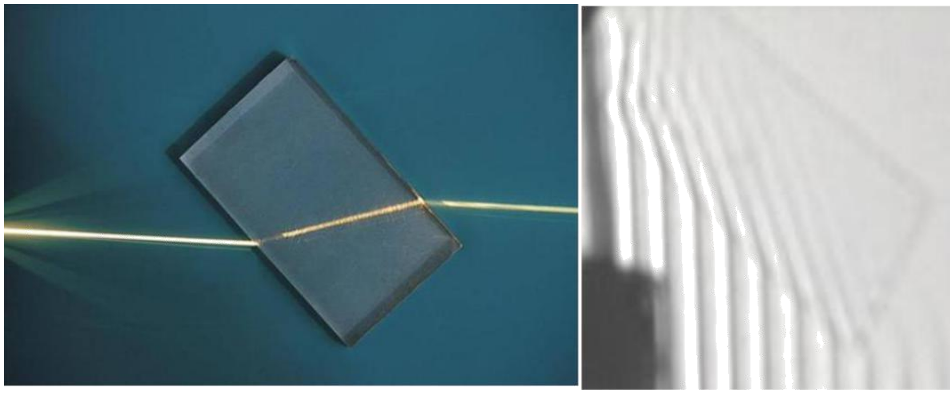
[10] Isaac Newton, Opticks, pg 370, (1730)
[11] John Tyndall, "Goethe's Farbenlehre," The Popular Science Monthly, pg 312, June 1880
[12] Augustine Ziggelaar, "How did the Wave Theory of Light take shape in the mind of Christiaan Huygens?" Annals of Science, 37 (2), pg 179, (1980).

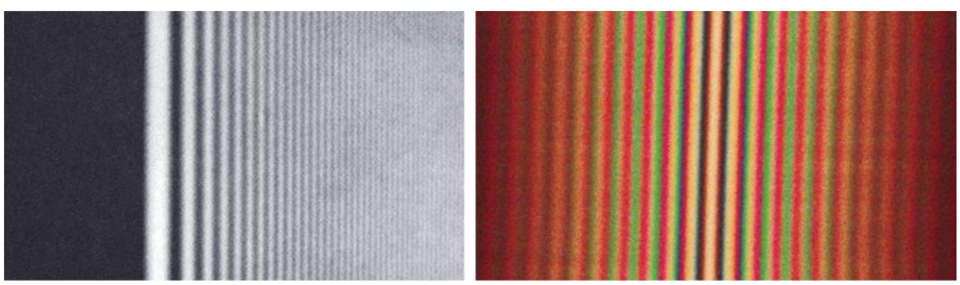


Refraction and its Wave Analogy

The first attempt by Huygens to bring this approach failed, due to Newton's "almost tyrannical authority" during the 18th century[13]. Thomas Young picked up on this thread, and drawing on his experience in music, applied his ideas about acoustics to optics, to translate sound into light[14]:

> … Young's awareness of sonic and musical phenomena prepared the ground for his work on light, right down to the details of the experiments that would finally satisfy Newton's demand that light be shown positively bending around obstacles.

This "bending" was observed near sharp edges and through small slits, and appeared to match the phenomena of waves quite well.



Light seen at the edge of a knife, and light on a screen after passing through two narrow slits

This led to the view of light as a wave in a fluid medium. However, another phenomenon was soon discovered, which led to a resurgence of the corpuscular approach: polarization. By noting that transmitted light gets extinguished when two Iceland Spar crystals are crossed at 90º with respect to each other, and passed fully when parallel, the concept of *turning* of light got added to the analysis. Since the corpuscle-supporters had been well versed with dealing with light as small balls, and Descartes himself had associated a spin to the balls, the immediate analysis of polarization was in terms of spinning balls. Biot, for example, spoke of "light molecules" that have a "rotary motion about their center of gravity" and that for the different colors, "the violet molecules (are) turning faster than the blue, the blues faster than the yellows, and so on down to the reds which will be the slowest of them all."[15] It is clear that light was seen literally as a solid rotating ball in such accounts – indistinguishable from a bag of

[13] Robert A Crone, A History of Color: the evolution of theories of lights and color. Dordrecht: Kluwer Academic Publishers. (1999) pg 78
[14] Peter Pesic, "Thomas Young's Musical Optics", OSIRIS 28, pg 15 (2013)
[15] Jean-Baptiste Biot, Recherches Expérimentales, 408 (1814)

marbles. This materialization of light had reached a point where Biot, in order to account for diffraction fringes that were described in terms of wave theory, had even said[16]:

> All the phenomena … could be represented with the most perfect fidelity by attributing to the light molecules two poles, one attractive, the other repulsive, which they would present alternatively to the surfaces of bodies, by turning with a uniform motion around their center of gravity. The light molecules and the surfaces of bodies would then be in the situation of two magnets which approach one another with their like and unlike poles.

The alternation between light and dark were hence attributed to the mechanism of magnetic attractions and repulsions. Note that this was nearly fifty years before Maxwell's theories of electromagnetism.

However, the wave theorists did not give up on polarization from their end. With Arago's support, Augustin-Jean Fresnel developed the wave equations to account for the straight-line appearance of light using the wave theory itself. He next moved on to the question polarization, where the wave had to be capable of "turning". He declared that the wave was transverse instead of longitudinal, so that it can now be turned at different angles! This was an astounding assumption, since waves in any medium are longitudinal, which act by compressions and rarefactions. *Transverse* waves only appear at the intersection of two media in different states, such as water and air. It was hard to imagine how these could ever be transverse, but Fresnel declared frankly to Poisson, who had raised some questions, that[17]:

> The equations of motion of elastic fluids, in which you think one has to find all the forms of vibration to which these are susceptible, are nothing more, at base, than mathematical abstractions.

However, Fresnel soon modified this position and attributed some peculiar properties to this medium in which his waves of light propagated, so that they can handle transverse waves. Thus the saga of the "ether" of the nineteenth century began, as an invented medium to "hold" light's vibrations.

When Faraday, the great cartographer of the electric and magnetic worlds, discovered that a "turning" of light occurred when it was passed through crystals in the presence of a magnetic field (Faraday rotation), it opened the door to relating the behavior of light to the magnetic field. When the ratio between certain types of electric units[18] was found to be close to the speed of light, Maxwell incorporated the *electro-magnetic field* into the description of light in his theory. When Hertz discovered that radiation from an electric spark induced another spark at a distance, and that the radiation showed many of the characteristics of light, the identification of light with the electromagnetic field wave appeared complete.

However, at the turn of the 20th century, phenomena of the photo-electric effect showed that the behavior of light was very similar to corpuscles once again! For instance, electricity excited in a material did not depend on the intensity of light at all, but only its intrinsic nature, such as color. The new "corpuscle" of sorts was called the photon, which has a "packet of energy" associated with it, and has some unique properties[19]:

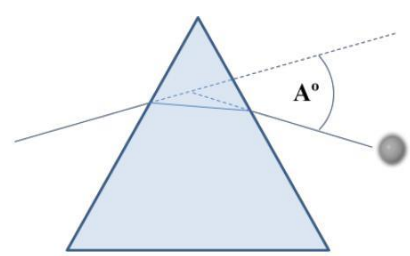
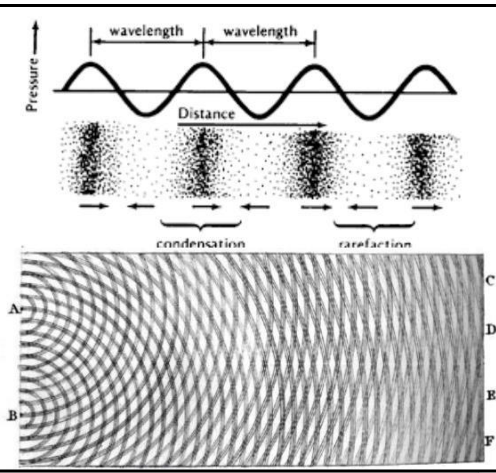
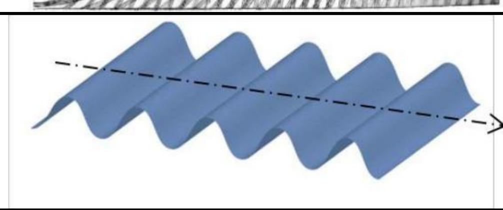
> … a photon representing a plane electromagnetic wave occupies the entire three-dimensional space… Also, it is known from quantum electrodynamics that … photons are not localized particles of light.

[16] Eugene Frankel, "Corpuscular Optics and the Wave Theory of Light," Social Studies of Science, 6, pg 141-84 (1976)
[17] Op. cit. 15
[18] 16 Wilhelm Weber, Rudolf Kohlrausch, "Ueber die Elektricitätsmenge, welche bei galvanischen Strömen durch den Querschnitt der Kette fliesst", Annalen der Physik, 99, pp. 10-25 (1856)
[19] Michael I.Mishchenko, "Gustav Mie and the fundamental concept of electromagnetic scattering by particles," Journal of Quantitative Spectroscopy & Radiative Transfer 110, pg 1210 (2009)

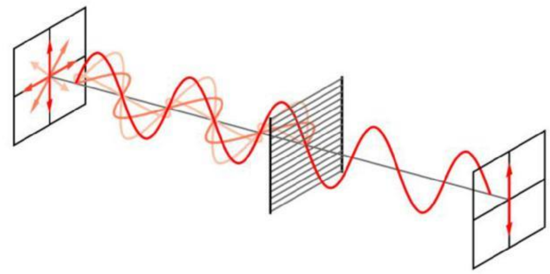
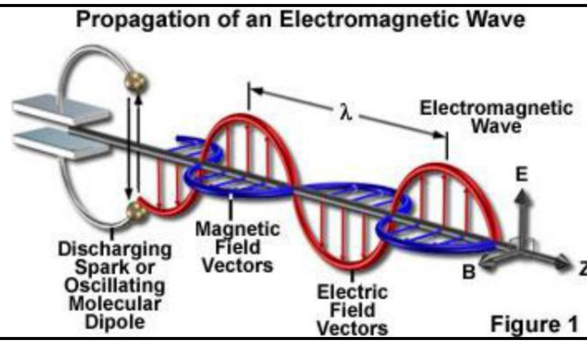
In other words, light has a discrete nature even if it is not a particle, and at the same time is seen as an electromagnetic wave. This new idea of light was incorporated into quantum theory as the famous wave-particle duality.

All in all, the journey of light has been a remarkable one: from being non-material to being associated with sound in air, waves in water, little rotating marbles, mathematical abstractions, waves in the mechanical ether, electro-magnetic field waves, and packets of energy. Mathematically as well, this journey has taken several parallel stages. In the case of the solid particle getting deflected by the prism, Newton associated a number with the angle of each color, calling it *degree of refrangibility*. When the wave theorists came on the scene, the number now became the *frequency* of a sinusoidal waveform that moves both in the positive and negative sense. Thus, the concept of *wavelength* was introduced. Upon further development, a phase (a shift in the wave in the direction of propagation) was also added to this description, which could only be represented using complex numbers in the wave equation[20].

These results can be combined in this way, in tabular form:

Idea of Light	Originators	Image	Mathematics
Solid	Descartes, Newton, Biot		Angular: A°
Liquid, Gas	Hooke, Huygens, Young		Scalar compression wave: asinθ, bcosθ
Liquid-Gas boundary, or 'ether'	Fresnel, Arago		Vectorial Transverse wave: Asinθ, Bcosθ

[20] Ricardo Karam, "Fresnel's original interpretation of complex numbers in 19th century optics," American Journal of Physics 86, 245 (2018)

Rotation of the field for polarization	Fresnel, Stokes, Jones		Polarization Matrix
Electromagnetic field	Maxwell, Hertz, Mie, CV Raman, until today		Complex wave: $\exp[i(kx–\omega t)]$

The action of light in a turbid medium is hence described today in terms of these electromagnetic waves scattering from minute particles.

This path to history brings us to crucial questions: Are all these iterations leading in the right direction? Does it lead closer to the nature of light and matter? And most importantly, are these approaches philosophically and logically consistent?

IV. Clarifying the nature of light

One common feature of the trend since Newton has been to attribute the nature of solids, liquids, gases, electric and magnetic fields to light. To check whether this is valid, the origin of this method has to be examined. Descriptions of light all have a couple of things in common: an experimental result, whether it is the separation of fringes of light or the deflection of a beam or the turning of the plane of polarization, can always be *numerically measured*. However, the process that is supposed to exist behind properties being measured are themselves neither *observable nor measurable*. For instance, the diameter or weight of the little "corpuscle" of light, the compression wave, the ether, the mechanical transverse wave, and the crests and troughs of the electromagnetic wave are *not observable*. What is the reason for this sustained use of imaginary "models" to account for the behavior of light? At the root of this approach is the separation of reality into two parts, one which has the 'primary qualities' which are supposed to be objective and one which has the 'secondary qualities,' which are supposed to be subjective. This split was clearly defined first by Galileo and extended by Newton:

> I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we locate them are concerned, and that they reside in consciousness. Hence if the living creature were removed, all these qualities would be wiped away and annihilated – Galileo Galilei , *The Assayer* (published 1623)

> For the rays, to speak properly, are not coloured. In them there is nothing else than a certain power and disposition to stir up a sensation of this or that colour. — Isaac Newton, Opticks (3rd ed. 1721, original in 1704)

The primary qualities – namely movement, force and size – are hence used to explain the secondary qualities – such as color. Therefore the 'explanation' of secondary phenomena, such as color, is attributed to primary phenomena such as oscillating waves or bouncing marbles or a combination of the two. However, since these phenomena are not observed, **they have to be imagined**! Rudolf Steiner pointed out this inconsistency very clearly in his writings on Goethean science[21]:

> Magnitude, shape, location, motion, force, etc., are perceptions in exactly the same sense as light, colors, sounds, odors, sensations of taste, warmth, cold, etc. Someone who isolates the magnitude of a thing from its other characteristics and looks at it by itself no longer has to do with a real thing, but only with an abstraction of the intellect. It is the most nonsensical thing imaginable to ascribe a different degree of reality to an abstraction drawn from sense perception than to a thing of sense perception itself. Spatial and temporal relationships have no advantage over other sense perceptions save their greater simplicity and surveyability. It is upon this simplicity and surveyability that the certainty of the mathematical sciences rests. When the modern view of nature traces all the processes of the corporeal world back to something that can be expressed mathematically and mechanically, it does so because the mathematical and the mechanical are easy and comfortable for our thinking to deal with. And human thinking does have an inclination toward being comfortable.

Right here is the crux of the matter: there is no logical reason to demote secondary qualities. Magnitude in the mathematical sense and color in the sense of the eye stand epistemologically on equal grounds of validity. As this separation between primary and secondary qualities by attributing different levels of reality to them is based on the reason of comfort or ease of calculation, it ceases to be a scientific principle in any sense. It is ironical that in the process of explaining facts of color with the images of oscillating waves, these “more real” waves have to be *imagined*. Even if it is argued that these images are simply useful tools to clarify understanding, one is once again encountering the same pitfall: whether one speaks of comfort or a “useful tool”, one is not speaking of consistent scientific methodology, but of a personal preference. The whole series of imaginary pictures, such as corpuscles, ether, waves and rotations of these waves are therefore a result of an *unscientific* approach to phenomena. This is why Steiner pointed to Goethe’s science as “being scientific in the truest sense”[22]. In the very act of positing unobservable images behind the observable process, the scientific process is derailed, with the result that it matters very little what understanding one has of the phenomena as long as some numbers can be calculated. Science degenerates to tricks of calculation.

The other issue with the properties attributed to light is that one is not very clear which properties belong to the substances that light is shining through, and which properties belong to the substance itself. When light displays fringes when passing past edges or through slits, is that a property of light, or the property of the geometrical object-setting? When the plane of polarization is altered due to a magnetic field, does that mean that light is magnetic, or that the magnetic field of the substance alters the light? Similarly, when an interference experiment is setup such that the screen is sensitive to electric fields[23], do the patterns in it show something about the nature of light, or the nature of the screen, or of the interaction? It is logically possible that a given property can be that of the light, that of the substance which light is passing through, or that of the interaction itself. For instance, when a finger is rubbed on the table and heat and sound are generated, that does not mean that the table and finger are “made up of heat” or “made up sound” – the heat and sound arise in the interaction. So, are all the properties being studied properties of the light, matter, or the interaction?

Historically, almost all properties of the phenomenon under observation have been imposed onto light, such that it is the carrier of the waves, the direction of polarization, the electric and magnetic responses, etc. Thus there is a twofold clarification required:

1. Make sense of the behavior of light without imposing unobservable pictures onto it.

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[21] Rudolf Steiner, “Goethe the Scientist,” Chapter XVII, pg. 247 (1950)
[22] Rudolf Steiner, “Wonders of the World, Ordeals of the Soul, and Revelations of the Spirit,” Lecture, August 28, 1911
[23] Otto Wiener, “Stehende Lichtwellen und die Schwingungsrichtung polarisirten Lichtes,” Ann. Phys. Chem. 38, 203-243 (1890)

2. Differentiate the properties of the medium and that of light

In order to do that, greater attention has to be paid to what is sensed, and what is conceptualized according to the sense data.

V. Light as a sensation and as a concept

To begin with, observation of the path of illumination of any light source at night shows that light itself is invisible, unless it interacts with matter in some way, be it dust or moisture or any other medium. This means one has to be careful in using the word light, which is commonly used synonymously with brightness, whiteness, brilliance, luminosity and so on, assuming visibility. Light as used in this scientific sense will refer only to something that is in itself invisible, without the intervention of matter. In order to specify this idea of it, light will be redefined in this sense. This light is different from the color that shows up when matter is interspersed and makes it observable to the eye. All colors, *including white*, are direct sensations of the eye, and are therefore expressions of light.

What about dark? One does not directly see dark, as a sensation of the eye. Hence, anything that does not activate the sensation in the eye, *even if it activates other sensory organs* is, for the purpose of the eye: *Dark*. Just as light is different from color, even if that color is white, dark is different from black. Dark is also invisible, but can arise visibly when a black image is presented say on a white background. It is also possible to observe dark arising when an illuminating source is blocked by opaque matter, i.e. as a shadow. Hence dark encompasses absence of even the possibility of sensation by the eye, blackness, and shadow.

Let us assume that when the sensation of color is obtained, and in order to 'explain' this sensation one seeks to draw a ray or wave-form. The image of a ray or a wave is only recognizable when there is a difference in color – say a black line on a white background. Thus, an 'explanation' needs the same perceptions as the perception that is supposedly being explained! One needs a black and white image in order to 'explain' black and white, which is the proverbial case of pulling oneself up by one's bootstraps. Drawing rays and waveforms to explain light is hence logically absurd. This entire process of image-making hence falls apart logically, and therefore the explanation or *concept* of color cannot be sought that way.

Since both light and dark are not directly perceptible, what are they in reality? It is clear that both of them *exist*, since the physical phenomenon corresponding to both ideas, such as white and black or bright and dull colors, are detectable. Since white and black cannot be 'explained' by means of any images, and one is speaking with respect to the sense of sight, light and dark themselves are the *concepts* that explain white and black. Light and dark encompass, but are not limited to, the sensation of white and black while being invisible at the same time. This is precisely because *their reality is as concepts or as ideas*, which can be converted into a sensation of white and black. Concepts as such are invisible. Light and dark are hence the concepts through which one 'explains' the world of the eye, the world of color. They are concepts incarnated in color, and as we shall see shortly, in matter.

What about the words "matter" and "object" that have been used so casually in the descriptions in the previous paragraphs? In terms of sense organs, one knows of what is called matter, whether gaseous, liquid, or solid with the help of the *sense of touch*. Even if smell and taste are also valid for matter, touch encompasses the very core of what we call mass or matter, and all the physical terms such as force, pressure etc. in the end reduce to this perception. Even air can be sensed if one waves the hand very fast. But the sense of touch is distinct from the sense of sight; one cannot 'see' a mass. It is impossible to tell, simply by looking, whether a rock is heavy or light – it has to be grasped and lifted. From the experience one might have had of banging into a glass pane that went unnoticed, it can become clear that the eye does not see pressure. Since pressure is perceived through the sense of touch, and we have previously noted that the concept dark includes all that is not sensed by the eye even if other senses can pick up on it, *matter* is in this sense also accurately termed dark.

Now, it is possible to combine the sense of touch and sight in order to obtain categories of matter. When something that one can touch is completely invisible, like most gases or even pure glass, they form transparent matter. At the other extreme, when no color is allowed to enter an object, it is opaque. When no color penetrates the surface of an object, but all of it is retained on the surface, this color is called *white*. A white object is perfectly opaque matter, as it takes on the color of the surrounding illumination and does not impose any change on it. A black object is not transparent, yet it is opaque in a different way: by trapping illumination. It arrests the colors and makes them disappear. Hence, white is *actively* opaque, gases and glass are *actively* transparent, and black is *passively* opaque.

There is also a peculiar property of light in contrast to matter is that experiments show that it manifests only as a speed – called the speed of light. This indicates that this word is not merely a noun, it is also a verb. It is important to keep this property in mind in further descriptions. Illuminating and darkening are the respective activities of light and dark. Therefore, when comparing activities, in the outer parts of the atmosphere where the gases thin out to almost nothing, light conquers dark, but is not visible since the gas offers no resistance. In a transparent medium light is active and *dominates* dark, which is only present as a non-visual sense of pressure. Since light dominates, through a transparent medium, the color white is seen as white and black is seen as black. In an opaque white medium, dark dominates light in the object's interior but gives white color to the surface. In a pitch-black opaque object, dark conquers light, rendering color inactive completely.

To summarize:

Gas-less space:	*Light* illuminates through and conquers *Dark*	Not perceptible to eye, not to touch
Transparent:	*Light* illuminates and dominates *Dark*	Less perceptible to eye, easy to touch
Opaque (white):	*Dark* darkens and dominates *Light*	Perceptible to eye and touch, eye dominates
Opaque (black):	*Dark* darkens and conquers *Light*	Perceptible to touch and eye, touch dominates

It might seem odd that in a white object *Dark* dominates, but in the general sense of *Dark* described here, and restricted as it is only to the sense of touch (interior of the object) and not as color, *Dark* is the right concept for it.

Between transparency and opaque (white), one sees the perceptibility to the eye increasing. Between opaque(white) and opaque (black), the activity of the eye is steady, but the variability is restricted to color. In order to study the interaction of the activity of the eye and the arising of color, in the sense of a verb, the right place to focus is where a medium is in between transparency and white opaqueness: **the semitransparent or turbid medium**.

Transparent medium, cloudy semitransparent media, white opaque object

This is the reason why Goethe focused on this type of medium as the right intermediary for the study of color. If he was to focus on light and dark simply as white and black colored surfaces, the logical choice would be to pick a gray surface. However, he was not content merely with colors as nouns, i.e. A is colored X and B is colored Y. He was more interested in colors as verbs, as to how they arise, how white *becomes* yellow, and how black *becomes* blue. This is in keeping with the active reality of illumination. It would therefore be illogical 'pin down' colors as nouns saying that Yellow = frequency one and Blue = frequency two. The semi-transparent medium is dark as much as it is perceptible to touch and is also beginning to oppose light through milkiness or cloudiness of the transparency. Therefore, the semitransparent medium is an indicator of this activity between being invisible and opaque.

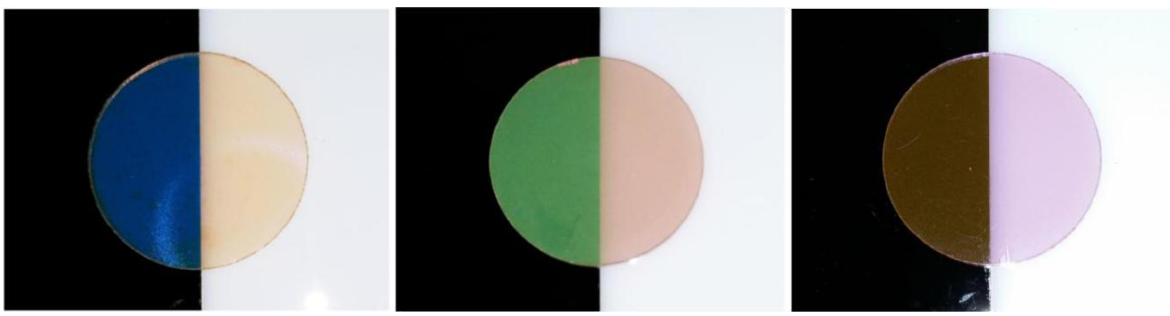
VI. Color Phenomena in Semitransparent Media: A Fresh Look.

By honing in on the activity of light and dark, choosing a semitransparent medium is mainly a matter of observing a solution that gradually turns fully white, and studying how illumination through it changes the color that appears. It is necessary to include light in its activity as an essential part of these experiments, so a bright source is used throughout. From the experiments with illumination of semitransparent media, the following observations are obtained as the fundamental activities of color:

The image of *Light* – white or illuminant – seen through a semitransparent medium becomes yellow.

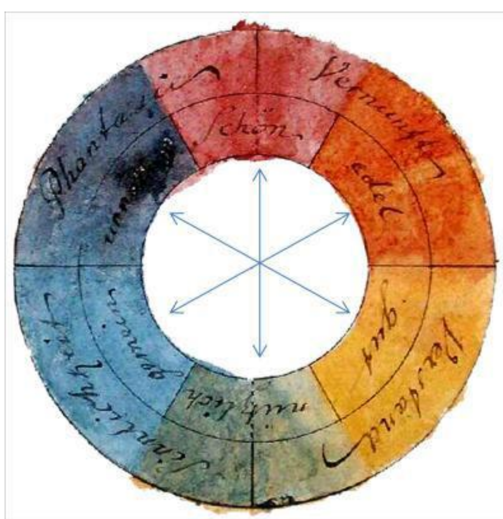
The image of *Dark* – black – seen through an illuminated semitransparent medium becomes blue.

After this, other color polarities can also obtained from colored semitransparent media. A simple example is actually seen in solutions with nanoparticles[24], where the same colored solution is seen on black and white:



Blue/Yellow-Orange Green/Magenta Yellow/Violet

From this, and several other experiments with after-images and colored shadows, the Goethean color circle was derived:



[24] https://nanocomposix.com/pages/color-engineering

This interaction of brightness and darkness through a cloudy medium was demonstrated in the 19th century by using the milkiness produced when sodium thiosulfate was reacted with sulphuric acid to obtain a gradually thickening milky white colloid made of sulfur suspension. This suspension was ideal to see the activity, since the milkiness increases with time. A white light source is passed through the solution. By understanding the relationship of turbid media to light sources, one would expect that the greater the darkening by the sulfur suspension, the redder the light source would appear, as seen through the solution. The solution itself, when seen in the dark, would appear blue.

White light source seen through a milky medium

And this is indeed the case. Following the experiments of Tyndall, Lord Rayleigh developed his theory of light scattering that is repeated to this day to "explain" the reason for the sky appearing blue: that the particles scatter away all the red. However, other experiments[25] showed something remarkable[26]:

> As is well known, the solution (which is at first perfectly transparent) becomes turbid when the particles form in it, and the transmission of light by the suspension gradually diminishes in intensity. The colour of the transmitted light, which is at first white, also changes, becoming, yellow, orange, red, and then deep crimson red. Finally, the solution (if in a sufficiently thick layer) becomes almost completely opaque. This had been previously supposed to terminate the sequence of phenomena. Keen and Porter observed, however, that after further lapse of time, light begins again to be transmitted by the suspension, the colour of the light which passes through being at first indigo, then blue, blue-green, greenish- yellow, and finally again white.

Here, when the darkening of the solution increases past a certain level, the role of brightness and darkness switch, and instead of seeing a light source through a semitransparent medium, one sees an illuminated medium in the darkness, leading to the reverse sequence of colors from violet to white, while the medium takes on a yellowish tinge. Particle size in such a process is not important by itself, but rather the *ratio* of darkness to brightness that the suspension generates as a whole.

It must be noted that Lord Rayleigh tried for years to solve this problem without success. CV Raman and Ray wrote two papers, with the results being explained by "permanent phase-relationship and consequent capacity for interference of the scattered and primary waves"[26,27]. It is also interesting to see that one never gets the full rainbow's colors (see section VII) as a sequence in these experiments with turbidity, which it should based on the extrapolation of Rayleigh's calculations.

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[25] B. A. Keen and A. W. Porter, "On the diffraction of light by particles comparable with the wavelength," Proc. R. Soc. London 89, 370-376 (1913, 1914) ; Abney, "Phil. Trans.," Part II, p. 653 (1880) ; W. Ritz, "Comptes Rendus," vol. 143, p. 167 (1906).

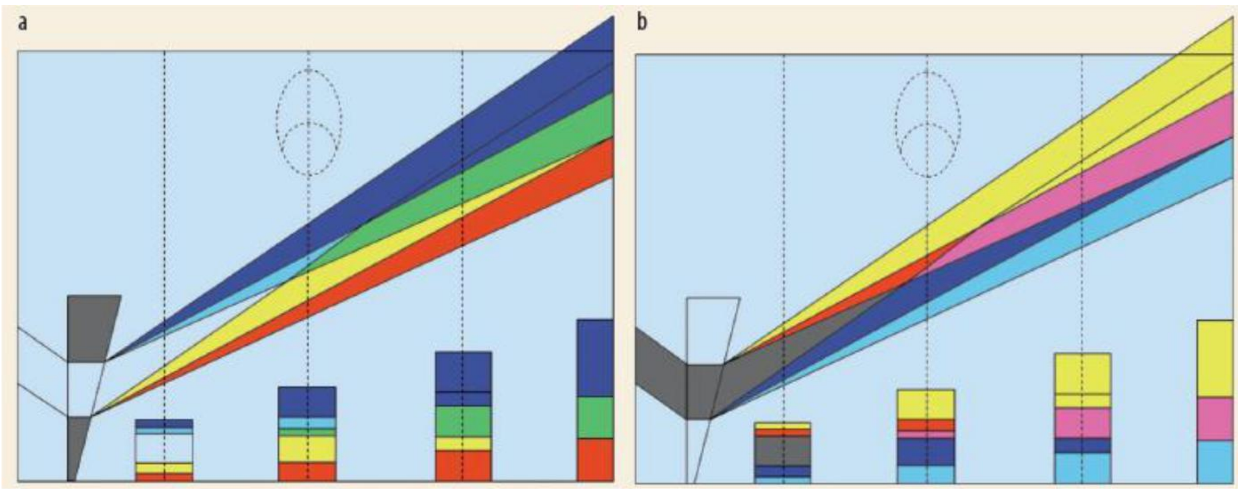
[26] C. V. Raman and B. Ray, "On the transmission colours of sulphur suspensions," Proc. R. Soc. London Ser. A 100, 102-109 (1921).

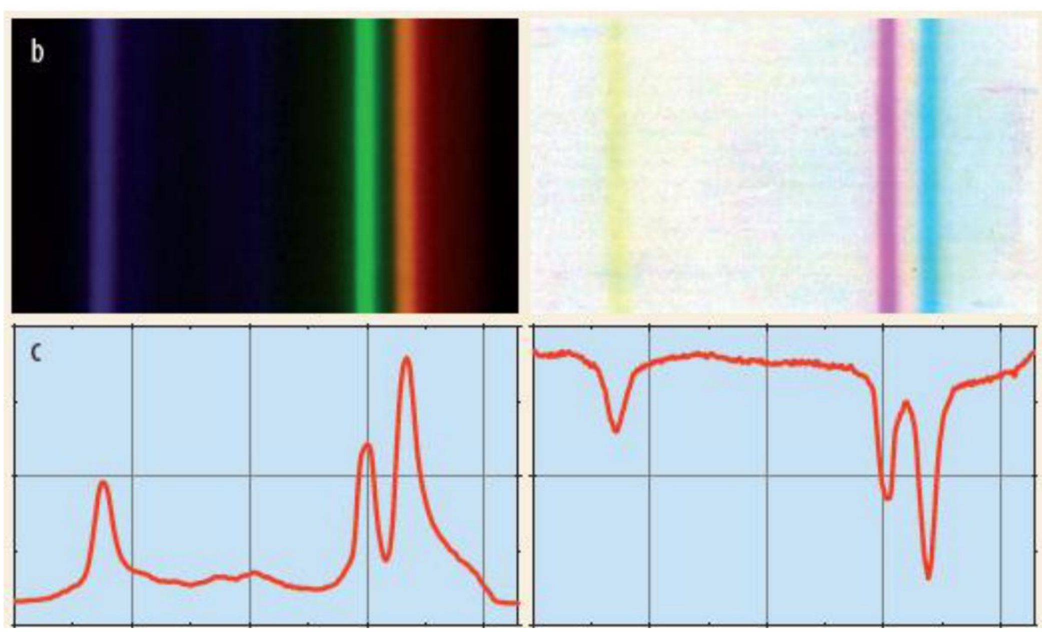
[27] B.B. Ray, "Scattering of light by sulphur suspensions," Proc. Indian Assoc. Cult. Sci. 7, 1 (1921).

Hence, the behavior of semitransparent media shows that, taken as phenomena of color alone, dark is active in the media with increasing turbidity, showing Goethe's relations of the Ur-phenomenon to be on a sound basis.

VII Spectra and Inverse Spectra

The polarity between *Light* and *Dark* is also brought out by several experiments with the Newtonian spectrum and the inverse Goethean spectrum forming two sides of the same coin. In the Newtonian spectrum, a white beam is shone through a prism (or grating) in a dark background, while in the inverse case a dark zone is passed through in a bright background. By inverting the "white slit in a black background" into a "black strip in a white background," the colors change completely. These experiments have been reproduced recently[28]:





These spectra show how problematic the concept of a single wavelength is, since the blue-violet line on Newton's spectrum corresponds to yellow line on the Goethean spectrum on the right. In order to be consistent, every "wavelength" must represent two colors. Yellow and blue-violet are opposites on the color circle. If, according to Newton, white light can be split into colors, by the same logic, darkness can also be split into their complementary

[28] Matthias Rang, Oliver Passon und Johannes Grebe-Ellis, "Optische Komplementarität Experimente zur Symmetrie spektraler Phänomene," Physik Journal 16 (2017) Nr. 3

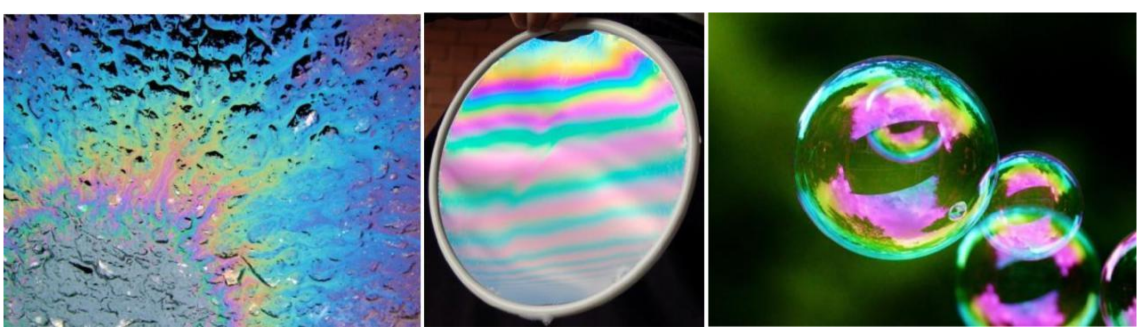
colors. It has been argued by Müller[29] in his theory of decomposition of dark rays that this darkness is a real presence and not merely the absence of light, since at the same place where one perceives heat in the Newtonian spectrum (infrared), one perceives coldness in the inverse spectrum (infra-turquoise).

The behavior of light is also distinctively opposite in both the spectra: in the traditional Newtonian case, green is formed at a certain distance onwards only, by the mixture of yellow and turquoise/cyan. In the inverse case, the mixture of red and blue/violet allows a brilliant magenta to appear after a particular point. In each case, the original spread of colors reduces to only three colors at a long enough distance: Red-Green-Blue in one case and Cyan-Magenta-Yellow in the other.

The arising of the Newtonian spectrum is predominant in the phenomena of the sky, when a light source or its reflection is being observed through a medium. The subject of rainbow formation due to a curved medium will be addressed in a different paper. Almost all the atmospheric phenomena like rainbows, glories, haloes, etc. show this spectrum quite clearly:

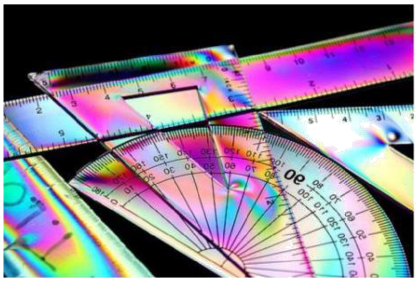
The Sun Rainbow and the Rainbow with inverted ordering seen at a waterfall from the top on a sunny day

Terrestrial phenomena, on the other hand, predominantly show the inverse spectrum. This is seen in soap bubbles, oil patches on the ground, thin films, and soap bubbles. These inverse rainbows can be identified by the distinct presence of magenta:

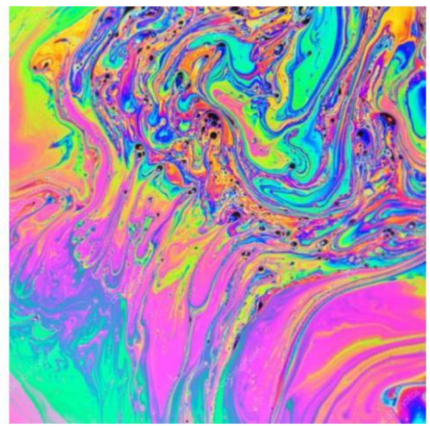


This inverse rainbow occurs with astonishing frequency around us, but the clear presence of magenta is not noticed and it is just called a “rainbow” instead of, say, “inverse rainbow”. With the closer affinity of this inverse rainbow to *Dark*, it is quite revealing to observe its presence in the study of matter through crossed polarizers, and even in the tempering of steel observed by Goethe as well:

[29] Olaf Müller, “Goethe’s Polarity of Light and Darkness,” J Gen Philos Sci https://doi.org/10.1007/s10838-017-9395-7 (online)

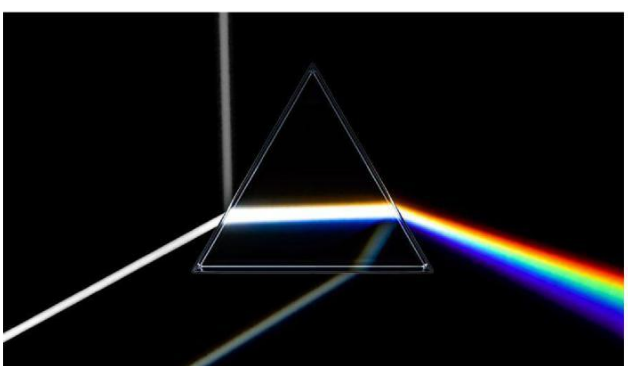
Couple of the most spectacular demonstrations of the inverse spectrum is the coating of quartz with a metallic sheen of vacuum-deposited gold (or other metals), and the colors of oil on water. A brilliant magenta flares up:

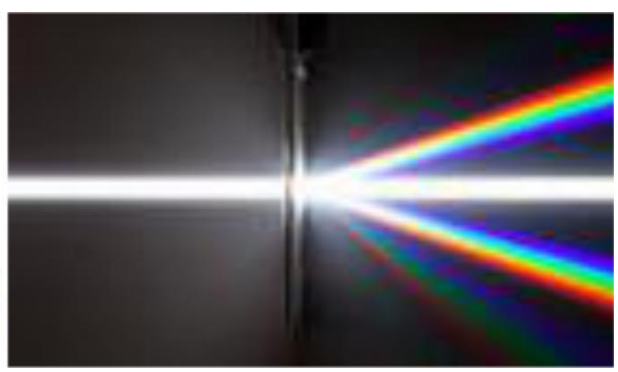
Hence, the abundant inverse spectra form the terrestrial pole of the formation of the rainbow, where dark is seen through an environment of light.

VIII. Spectrum formation in Prism and Grating

Nowadays, prisms are not used as much as diffraction gratings, since it was seen that they do not vary based on materials as much as prisms do. However, this does not change anything with regard to spectrum formation, since both the prism and the grating have white light in the middle, and only produce green at a distance:



Prism



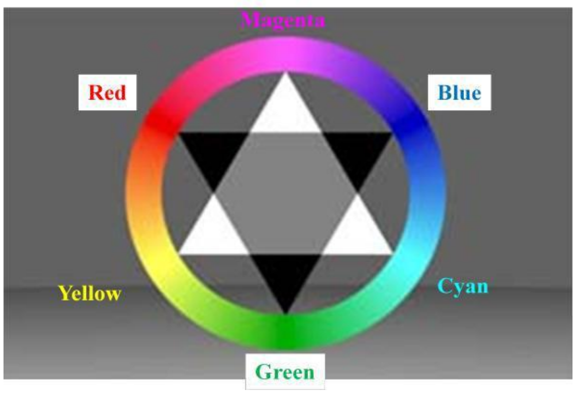
Grating

It is only by conducting experiments at a far off distance from the prism or grating that the current spectroscopic instruments manage to obtain their spectra without the white in the middle.

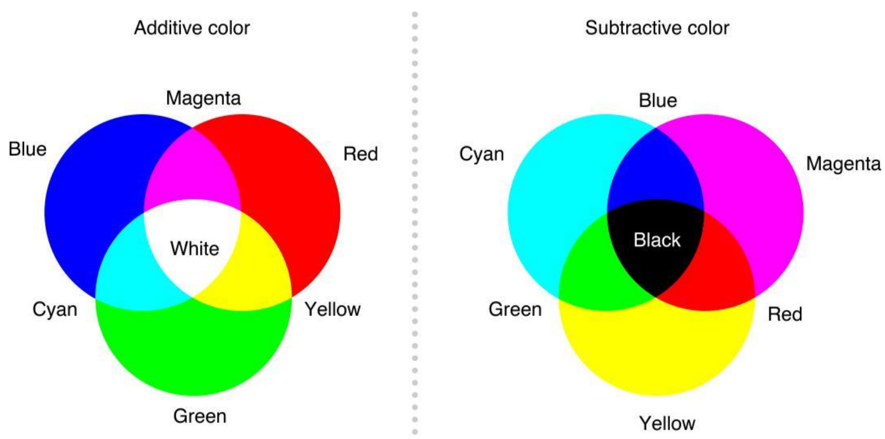
A similar process occurs with the inverse spectrum as well. However, the idea of "decomposing" darkness into its constituent "dark rays" (footnote 27) suffers from the same logical fallacies of the decomposition of white light by Newton. Not only is light incompatible with the corpuscular ideas of decomposition, but the *arising* of green and magenta at a specific distance from the prism or grating removes all justification for either of these to be present in the light or dark beams as "constituents". Once again, nouns and verbs are confused. The polarity of *Light* and *Dark* shows that one cannot choose one spectrum over the other, or even both of them simply put together as the accurate way to describe reality. The existence of two polar opposites directs us to the middle position that is beyond either one of them, but which can still combine the opposites into a harmonic whole. This position provides an opportunity to introduce mathematics into the theory of color in a completely different qualitative sense.

IX. Mathematics of Color

In the traditional sense, mathematics are applied mainly for quantities, but not usually for qualities. For instance, one can adjust the brightness of a screen up and down a number of notches, but that is different from associating a number for the qualitative aspects of color. In order to determine a mathematical relation *within* the interrelationships of color rather than one imposed on them from without, it is of foremost importance to study them in their own element. This is best done by laying out the color wheel of Goethe in a more contemporary form:



This color wheel is well known in its two aspects especially with the rise of technology in its two applications: images on screens and on printed material. All screens, such as television, smartphone, or computer screens, use the additive color process of red, green and blue *lights* giving white. Printing utilizes the magenta, cyan and yellow *pigments or inks* to give black.



It can be seen that in the case of additive mixing with lights, blue and yellow, red and cyan, green and magenta form opposites, and they combine to give white. This is the primary polarity identified by Goethe among the opposite colors in his color wheel. These same combinations in pigments give black. Gray is conspicuous by its absence.

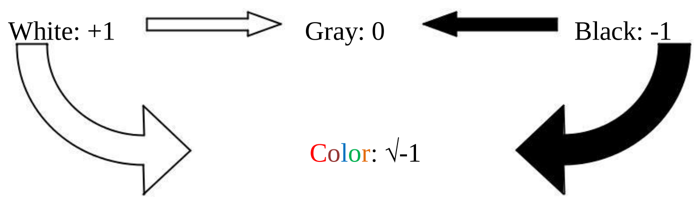
Now, let us observe the behavior of these colors one by one. White is the direct representative of light, as black is the representative of dark. Other colors that arise have brightness or darkness that is in between that of black and white. As colors appear to the eye, cyan, magenta and yellow appear brighter than red, green and blue. Red and blue appear darker than green, yellow and cyan appear lighter than magenta. This polarity shows that at the root of this entire process, white and black show the extreme poles that can be represented by positive and negative:

White: +1 Black: -1

Here the number 1 is to be seen as a quality, not a quantity. If it is seen as a quantity, white and black have to be attributed 1 and 0, as is done in the brightness settings of all screens, or in even simpler terms, a bulb being turned on and off. The quality of light is therefore +1, and is expressed as such in white. Dark is -1, expressing itself in black. If there is a direct "intermixing" of light and dark, one gets the value 0, i.e. colorless gray.

White: +1 ⇒ Gray: 0 ⟵ Black: -1

Is there another way that +1 and -1 can be interrelated, without the direct annihilation represented by 0? In mathematics, there is the concept of the imaginary number, which has troubled mathematicians for nearly a couple of centuries, as they struggled to discover its physical meaning. It is expressed as the square root of -1, or √-1. This provides the alternative route to the interactions of *Light* and *Dark*, so that it can give rise to *color*:



Imaginary numbers are in the "middle" of black and white, while retaining the qualities of *Light* and *Dark*. It is for this reason that Rudolf Steiner, who mastered and carried forward the ideas of Goethe, suggested that colors be studied using these numbers:

> At this point you have to apply to the light effects a set of facts which today are only vaguely felt and not by any means explained, namely the relation between positive and negative numbers and imaginary numbers.[30]

This is precisely the relation being elaborated here. The imaginary number also solves the mystery of why Goethe chose the odd concept of the turbid or semitransparent medium. When the activities of *Light* and *Dark* are balanced, one has both transparency and opaqueness possible, just as it is in such a semitransparent medium. Lacking a familiarity with formal mathematics, while at the same appreciative of the mathematical method of seeing phenomena, Goethe could not reach the point of identifying color as related to imaginary number, and therefore only hinted at the concept through his *trübes mittel*. It also shows why, when Newton painted the colors of the rainbow on a disk and spun it, he obtained gray instead of white. When the effects of the colors are neutralized, only gray is the remnant. This provides additional proof of the incompatibility of the "decomposition" of white light or the "composition" of colors into white.

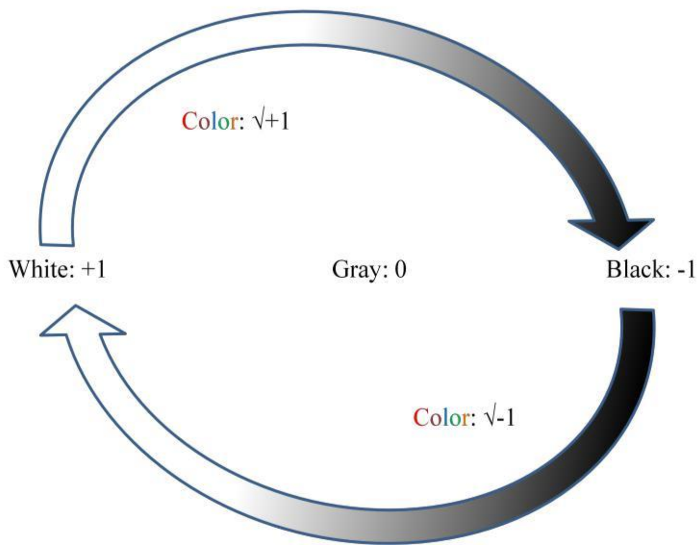
[30] Rudolf Steiner, Warmth Course, Lecture XII, March 12, 1920

Moving ahead with this, what is the nature of the individual colors? In order to attribute individual colors, one needs to use three sets of polar imaginary numbers. Such a set was already discovered by the mathematician William Hamilton, in his quaternion algebra, as the three numbers *i*, *j*, and *k*. They also have the following interrelations:

i.i = *j.j* = *k.k* = -1, since they are all imaginary

i.j = *k*, *j.k* = *i*, *k.i* = *j*, and *i.j.k* = -1

Note that Hamilton's expressions cannot be simply "downloaded", as one must see if it incorporates all of the reality of color. Each color is between light and dark, therefore it can act either in its lightening capacity or its darkening capacity. The same red, as a *verb* and not as a noun, can act as a light source or a pigment. So, two types of "*i*" are required. In algebra, these are the roots of both +1 and -1, and the roots √+1 form what is known as "split complex algebra," and the roots √-1 form the regular complex algebra. These expressions can be borrowed here, and applied in their qualitative sense. That means the number 1 in +1, -1, √+1 and √-1 does not represent a magnitude 1 but simple serves as an indicator of the quality of positive, negative, and imaginary. It can be represented like this:



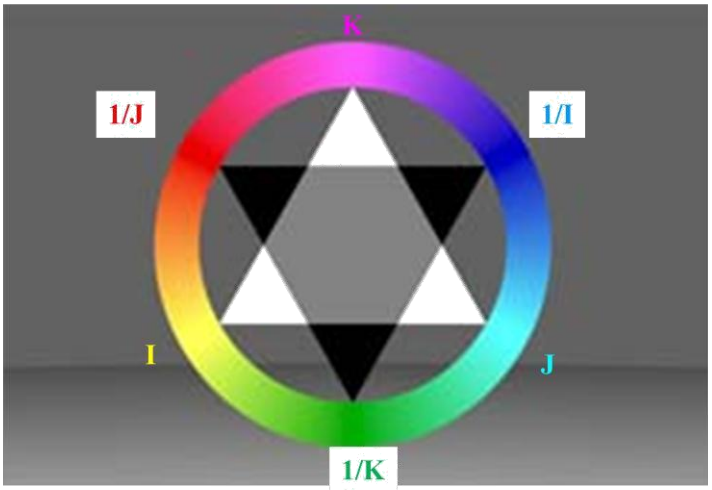
Let us use upper case to denote the lightening (additive and√-1) capacity and bars above upper case letters to show darkening (subtractive and √+1) capacity. Also, multiplication has to be seen as a combination, and not as the usual multiplication of magnitudes. For example, if R stands for red light and G stands for green light, R.G is the light that is formed by shining both together. With this in mind, the follow system is revealed:

Additive Colors (Lightening)

$$I.I = J.J = K.K = -1, \qquad and\ \frac{1}{I} = -I\ etc.$$

$$-I.-J = -J.-I = K, -J.-K = -K.-J = I, -K.-I = -I.-K = J$$

$$I.J = J.I = -K, \qquad J.K = K.J = -I, \ \ K.I = I.K = -J, and\ I.J.K = +1$$



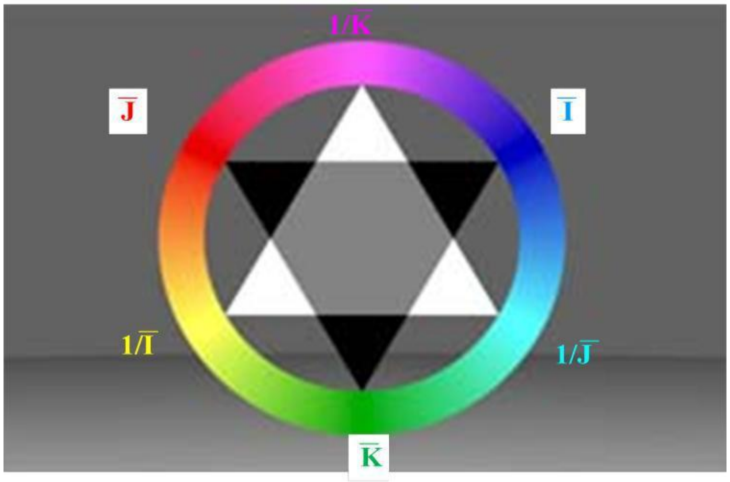
All three colors (CMY) together give white (+1).

Subtractive Colors (Darkening)

$$\bar{I}.\bar{I} = \bar{J}.\bar{J} = \bar{K}.\bar{K} = +1, \quad and\ \frac{1}{\bar{I}} = \bar{I}\ etc.$$

$$-\bar{I}.-\bar{J} = -\bar{J}.-\bar{I} = \bar{K}, -\bar{J}.-\bar{K} = -\bar{K}.-\bar{J} = \bar{I}, -\bar{K}.-\bar{I} = -\bar{I}.-\bar{K} = \bar{J}$$

$$\bar{I}.\bar{J} = \bar{J}.\bar{I} = -\bar{K}, \quad \bar{J}.\bar{K} = \bar{K}.\bar{J} = -\bar{I}, \ \bar{K}.\bar{I} = \bar{I}.\bar{K} = -\bar{J}, and\ \bar{I}.\bar{J}.\bar{K} = -1$$



All three colors together (RGB) together give black (-1).

The behavior of the colors show that the different elements are commutative i.e. a patch of yellow light lit upon by blue light has the same color as the patch of blue light lit up by the yellow light.

Two things must be made extremely clear: the descriptions in this section are mainly exploratory, in order to show the direction in which color studies can be done with mathematical precision of a qualitative kind. Further research is necessary to study this further. Secondly, at no point must these mathematical expressions be allowed to fall back into regular algebraic forms, which will inevitably add in many assumptions. For instance, in terms of the lightening colors, yellow = I, and red = 1/J or –J. In actual practice, yellow becomes red through the action of the medium, therefore I *becomes* –J. Similarly blue (1/I or –I) becomes cyan (J) through the action of an illuminated medium. The possibilities of turning the axes such that I ⇒ –J or J ⇒ –I may look like sacrilege to algebraists. However, such transformations have to be introduced in order to stay true to the phenomena of color.

It can be argued that by expressing the relations of colors in this way, nothing new is gained, since the mathematics is not used for calculations. In order to examine that, the relations used above can be summarized like this:

1. By ignoring all the rest, and focusing only on 1 as a magnitude, one gets the *corpuscular viewpoint*.

2. By focusing on the interchange of +1 and -1, one gets the basis for the *wave equations* used in interference and diffraction.

3. By focusing on √-1 (or *I*) and the fact that *I.I* = -1, one gets the basis for *color annihilating color*, which is at the basis of light cancellation in both interference and diffraction.

4. By focusing on √+1 (or Ī) and the fact that Ī.Ī= +1, one gets the basis for pigment enhancing pigment, which is at the basis of the action of paints, as well as constructive interference.

5. By focusing on the relations of *I* becoming -*J* and *J* becoming -*I*, as to how they are 90º away from each other, one gets to the basis of *polarization*, *phase changes* and the phenomenon of rotation of the plane of polarization. Complementary colors in the case of double refraction also arise the same way, which will be analyzed in a separate paper.

5. By comparing the behavior of the imaginary number with conventional equations, for instance as exponentials (exp (ikx)), one can find that this inclusion is at the root of why the mathematics works as it does. For example in Mie theory, as well as in modern material science, complex numbers for electric and magnetic permeabilities, as well as for refractive indices, without a clear justification. That justification is obtained here.

6. By observing that the imaginary number, when multiplied, gives +1 and -1, as well as simply the magnitude 1, the *true root of the wave-particle duality* is found.

7. By observing the bipolar nature of the imaginary numbers, the odd category of semitransparent medium becomes directly relevant, since it accurately displays this half-way nature.

Hence, relating the imaginary numbers systematically to the behavior of color sheds new light on a host of phenomena that are usually unclear, or require a lot of calculations to make sense of. Most important of all, one is throughout taking care not to impose the mathematics *on to* the behavior of colors, but to reveal the mathematical qualities already existing in the color relationships.

X. Concluding remarks

By examining the inherent assumptions that usually play a role in color phenomena, it was found that in most cases qualities are attributed to light that do not belong to light, both logically and phenomenologically. By clearing away the assumptions, examination of basic color phenomena shows a polarity of *light* and *dark*, and the turbid medium is a result of the ratio of action of light and dark. This polarity is also reflected in the existence of two polar spectra: one inverse of the other. This also implied two colors for each "wavelength", or position on the spectrum. The Newtonian spectrum, found mostly in the sky, becomes RGB at a distance, while the inverse spectrum, mostly terrestrial, becomes CMY at a distance. Choosing either one of them as the basis for the other becomes intrinsically biased, hence a way was found to find a way through these opposing polarities, leading to the idea of the imaginary numbers and a way to treat colors using qualitative mathematics. Obviously, this is merely a beginning of research into this subject. Several phenomena, such as the details of polarization, the origin of diffraction patterns, and the formation of rainbows, can be elaborated on this basis. These subjects are set aside for a future paper.